How much will we learn from the CMB ?

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Abstract. The purpose of this article is to give a brief account of what we hope to learn from the future CMB experiments, essentially from the point of view of primordial cosmology. After recalling what we have already learnt, the principles of parameter extraction from the data are summarized. The discussion is then devoted to the information we could gain about the early universe, in the framework of the inflationary scenario, or in more exotic scenarios like brane cosmology.

1. What we have learnt already

I will start by recalling briefly what we have already learnt from the Cosmic Microwave Background (CMB), which is already in itself rather impressive. For more details, the reader is invited to refer to the numerous reviews on the subject, for example the lecture notes of the 1993 les Houches school by R. Bond [1] and those of the 1999 les Houches school by F. Bouchet, J.L. Puget and J.M. Lamarre [2].

As is well known, the CMB was predicted in 1948 [3] and discovered less than twenty years later [4]. Since then, it has been measured repeatedly, and with increasing precision. The CMB spectrum is that of a black body to a very high precision. This feature gives one of the strongest arguments in favour of the hot big bang model, according to which the photons where in thermal equilibrium in the past. This also implies some very stringent constraints on energy release in the universe after a redshift $z \sim 10^{6-7}$.

A second important feature of the CMB is that it is almost isotropic but not quite. There is first a dipole at the 10^{-3} level, which is usually interpreted as the motion of Earth with respect to the CMB rest frame. There are then higher multipole anisotropies at the 10^{-5} level, which had been expected for a long time, and observed for the first time by the COBE satellite ten years ago [5]. These anisotropies had been expected for a long time because they were believed (and are still today) to be generated at the moment of last scattering by very tiny cosmological fluctuations, the ancestors of the present cosmological structures.

Before going on, let us recall quickly the basic formalism to describe the CMB anisotropies. The temperature anisotropies can be expanded in terms of (scalar) spherical harmonics,

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi). \tag{1}$$

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For the theorist, the temperature is an isotropic random field and therefore the multipole coefficients a_{lm} random variables. One can define the angular power spectrum by

$$C_l = \langle |a_{lm}|^2 \rangle, \tag{2}$$

which is enough to specify entirely the temperature random field if it is Gaussian.

What is usually plotted is the quantity $l(l+1)C_l$. If one assumes coherent scale-invariant initial fluctuations, such as those produced by inflation, one expects to observe a plateau at low l, corresponding to large angular scales. At smaller scales, one expects to see oscillations [6]. The reason is that a given Fourier mode, characterized by a constant comoving wavenumber k, will start to oscillate as soon as its wavelength is within the Hubble radius, i.e. when pressure enters into play. Of course, different Fourier modes enter the Hubble radius at different times and thus, at a given time, they are at different stages of their oscillatory pattern. The CMB, being then essentially a snapshot of the last scattering surface, we thus expect to see oscillations in the angular power spectrum.

By contrast, the topological defect models, the main competitor facing inflation for many years, do not predict oscillations because the contributions of many incoherent fluctuations, generated at different times by topological defects, add and smear out the oscillations [7].

With the recent data from Boomerang [8], Maxima [9] and DASI [10], the picture which is now emerging is that predicted by inflation and not by topological defects, which cannot account for the main part of the initial spectrum. Moreover the position of the first peak suggests that our Universe is quasi-flat.

In less than ten years, we have thus learnt a lot from the CMB anisotropies. The coming decade should be extremely fruitful as well, with several planned experiments, the most ambitious being the Planck satellite mission [11]. So far, it is however remarkable, for cosmology, that there has been no real surprise (apart a quantitative surprise with the amplitude of the anisotropies) with the CMB. The simplest theoretical models were able to predict in advance what we have observed. The question is how long this situation is going to last. With the increasing precision of the forthcoming experiments, will the simplest early universe models survive or will one need slightly more complicated models, a lot of which have already been explored by theorists ?

2. What we hope to learn about the cosmological parameters

Before discussing the extraction of cosmological parameters from the CMB data, it is important to recall that the actual CMB signal is the sum of three components:

- primary anisotropies
- secondary anisotropies
- astrophysical foregrounds.

The separation of these three components will involve a lot of work in data analysis as well as some understanding of the physical processes producing these components. Although in this review I will focus on the early universe perspective, for wich the relevant information comes from the primary anisotropies, it is worth emphasizing that the other components also provide useful information in various fields of cosmology and astrophysics. For instance, the Sunyaev-Zeldovich effect corresponding to the

scattering of CMB photons from hot gas in galaxy clusters, provides an important proble of the physics of clusters.

Let us now concentrate on the primary anisotropies. For a scale-invariant initial spectrum, the expected spectrum today is composed of a plateau at small l and of a succession of acoustic peaks at larger l, as mentioned before. What is important for cosmology is that the height and angular position of the acoustic peaks are directly related to the parameters of the cosmological model. In fact it makes more sense to divide this set of parameters into two classes:

- the parameters that define the geometry and matter content of the universe: this in general includes the Hubble parameter H, the spatial curvature Ω_k, the total energy density Ω, the baryonic contribution Ω_b, the cosmological constant Ω_Λ (this could be replaced by a time-dependent quintessence type matter component), and the cold dark matter contribution Ω_c, where we have used the generic notation $\Omega_X = 8\pi G\rho_X/3H^2$ for any component with energy density ρ_X;
- the parameters that define the "initial" fluctuation spectra, i.e. the spectra produced during the early universe: this usually includes the normalisation of the scalar spectrum, its index n_s, the tensor spectrum index n_t and the tensor/scalar amplitude ratio.

There is a difference of nature between these two sets of parameters in the usual framework of perturbed FLRW cosmology. The first set of parameters is associated with the homogenous part of cosmology (there might be some unknown in the total number of components of dark matter) whereas the second set of parameters is aimed at parametrizing spectra, i.e. functions. It is preferable to reserve the name of 'cosmological parameters' to only the first set and to denote the second set by 'initial perturbation' parameters to insist on their different nature.

One of the present quest of cosmology is to determine, with the highest possible precision, the value of the cosmological parameters (in the restrictive sense) from the CMB data to come. One must be aware that the "measurement" of these cosmological parameters, as well as the expected precision, can depend sensitively on the assumptions concerning the initial perturbation spectra and their parametrization. Reversing the perspective, it will be indispensable to combine CMB data with other measurements of the cosmological parameters in order to get as much information as possible on the initial perturbation spectra.

It is now instructive to recall the principle of the extraction of the cosmological parameters from the CMB data. Let us start from a CMB map (which assumes a lot of work in data analysis has already been done), which we call 'D' (for data). Let us call the set of cosmological parameters we wish to extract 'T' (for theory).

The idea is to use Baye's theorem, which simply expresses in two ways the joint probability to have both D and T:

$$P(T|D)P(D) = P(D|T)P(T), \tag{3}$$

where $P(D|T)$ is usually called the likelihood and denoted $L(D|T)$. The method is then to maximize the likelihood function, given the data, and thus to obtain some estimates $\hat{T}(D)$ of the parameters. Expanding $\mathcal{L} = -\ln L$ in the neighbourhood of $\hat{T}$, L can be approximated by a Gaussian distribution. The expectation value of the inverse covariance matrix

$$F_{ij} = \langle \frac{\partial^2 \mathcal{L}}{\partial T_i \partial T_j} \rangle, \tag{4}$$

called the Fisher information matrix is now familiar in papers dealing with the cosmological parameter estimation from CMB data, and expresses how much the likelihood is peaked around the best estimate. Assuming the CMB and the noise fluctuations to be Gaussian, the Fisher information matrix is given by

$$F_{ij} = \sum_l \frac{2l+1}{2} \left\{ C_l + \frac{4\pi\sigma^2}{N_p} \exp\left[\theta_b^2 l(l+1)\right] \right\}^{-2} \frac{\partial C_l}{\partial T_i} \frac{\partial C_l}{\partial T_j}, \tag{5}$$

where the additional exponential term comes from the beam smearing and its multiplicative coefficient from the instrumental noise.

Let us summarize the limitations to the determination of the cosmological parameters from the CMB data. They can be classified in three categories:

- Cosmic variance: this comes from the fact that we are observing a limited number of realizations of a random field. The cosmic variance is bigger at large angles and depends on the sky coverage for a given experiment.
- Degeneracies between cosmological parameters: one gets a degeneracy when different sets of cosmological parameters reproduce the same anisotropy spectrum. A well-known example is the deneracy corresponding to a variation of Ω_k and Ω_Λ such that the angular size distance to the surface of last scattering remains constant ($\Omega_b h^2$, $\Omega_c h^2$ fixed). This problem of degeneracies can in general be overcome by resorting to other types of cosmological observations, such as the supernova surveys or the large scale structure surveys.
- Degeneracies between cosmological parameters and the primordial spectra: this is the case when a variation of a cosmological parameter can be mimicked by a modification of the primordial fluctuation spectra.

Of course, additional limitations come from the substraction of the foregrounds from the CMB maps.

3. What we hope to learn about the early universe (in the inflationary paradigm)

Beyond the possibility of measuring the cosmological parameters, cross-checking with other cosmological observations, the CMB contains the extraordinary perspective to tell us something about the early universe. One should be very cautious here, and not overstate the possibility to 'see' early universe physics in the CMB. What the CMB will provide is a consistency check for any early universe model. Indeed, any early universe model, which explains in an unambiguous way how the primordial fluctuations are generated, can be confronted with the CMB observations, and either be rejected or be declared compatible with the data (with some constraints on the parameters of the model). However, what remains unknown to us is the amount of degeneracy among the early universe models themselves, that is how many consistent early universe models can reproduce the same spectrum of cosmological perturbations. This applies in particular to the question of proving or disproving inflation with the CMB data: the CMB data can tell us if the cosmological perturbations in the early universe follow a quasi-scale invariant spectrum; they cannot tell us directly if the actual mechanism which produced these fluctuations is really the gravitational amplification of the quantum fluctuations of a scalar field during a slow-roll phase. It is therefore always a healthy procedure in primordial cosmology to try to find alternative

models, based on different principles, with the condition of course that they must be compatible with the data.

This being said, let us now review what the CMB could tell us, if we believe that inflation is the correct description of the early universe (for more details, see the reviews by Mukhanov, Feldman and Brandenberger [12] and by Liddle and Lyth [13]). The simplest models of inflation are based on a single scalar field, ϕ, with a potential $V(\phi)$. The phase of interest is the so-called slow rolling regime, when the scalar field is moving slowly, and which is characterized by the two conditions

$$\epsilon \equiv \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \quad \eta \equiv m_P^2\left[\frac{V''}{V} - \frac{1}{2}\left(\frac{V'}{V}\right)^2\right] \ll 1, \tag{6}$$

where $m_P \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass. In the slow-roll regime, the homogeneous equations of motion for the scalar field reduce to

$$3H\dot{\phi} \simeq -V', \quad 3H^2 \simeq \frac{V}{m_P^2}. \tag{7}$$

During the inflationary phase, the scalar fluctuations of the metric can be written, in an appropriate coordinate system, in the form

$$ds^2 = a^2\left[-(1+2\Phi)d\tau^2 + (1-2\Phi)\,d\vec{x}^2\right]. \tag{8}$$

Defining the correlation spectrum $\mathcal{P}_\Phi(k)$, in Fourier space, by

$$\langle \Phi_{\vec{k}}\Phi^*_{\vec{k}'}\rangle = 2\pi^2 k^{-3}\mathcal{P}_\Phi(k)\delta(\vec{k}-\vec{k}'), \tag{9}$$

the quantum fluctuations of the inflaton field, initially in their vacuum, can be shown to produce a fluctuation spectrum given by

$$\mathcal{P}_\Phi(k) = \left[\left(\frac{H^2}{\dot{\phi}^2}\right)\left(\frac{H}{2\pi}\right)^2\right]_{k=aH} = \frac{128\pi}{3}G^3\left(\frac{V^3}{V'^2}\right)_{k=aH}, \tag{10}$$

where the subscript means that the various quantities are evaluated at Hubble radius crossing, i.e. $k = aH$, for a scale k. This spectrum (up to an overall constant according to the various definitions in the literature) is called the *scalar spectrum*. It is quasi-scale invariant since the scalar field, and thus the value of the potential and of its derivative, are supposed to vary slowly during this inflationary phase. One can also evaluate the scalar spectrum index $n_s(k)$ (which is weakly scale dependent for a slow roll regime):

$$n_s(k) - 1 \equiv \frac{d\ln\mathcal{P}_\Phi(k)}{d\ln k} = 2\eta - 4\epsilon. \tag{11}$$

In addition to the scalar spectrum, the inflationary phase will also generate a spectrum of gravitational waves. Gravitational waves are perturbations of the metric of the form

$$ds^2 = a^2\left[\eta_{\mu\nu} + h^{TT}_{\mu\nu}\right], \tag{12}$$

where $h^{TT}_{\mu\nu}$ is a traceless transverse tensor. They possess only two physical degrees of freedom (the two polarizations $+$ and $\times$), which can be described effectively as two scalar fields

$$\tilde{\phi}_{+,\times} = \frac{h_{+,\times}}{\sqrt{32\pi G}}. \tag{13}$$

The spectrum of gravitational waves, or *tensor spectrum*, is then simply given by

$$\mathcal{P}_g(k) = 2 \times 32\pi G \times \left(\frac{H}{2\pi}\right)^2 . \tag{14}$$

As for the scalar spectrum, one can define a tensor spectral index,

$$n_t(k) \equiv \frac{d \ln \mathcal{P}_g(k)}{d \ln k} = -2\epsilon. \tag{15}$$

An important consequence of the above results is that the ratio of the tensor and scalar amplitudes, $\mathcal{P}_g/\mathcal{P}_\Phi$ is proportional to $(V'/V)^2$, i.e. proportional to ϵ. This implies that if one was able to measure the scalar amplitude as well as the tensor amplitude and tensor index, then one could *check* if this consistency relation is satisfied. This would probably represent one of the most significant tests for inflation.

After this general introduction to inflation, let us just present the three main categories of models (many more details on the numerous models of inflation and their link with particle physics can be found in a recent review by Lyth and Riotto [14]) .

3.1. Chaotic type models ($0 < \eta \le \epsilon$*)*

This category corresponds to models with a scalar field amplitude of the order of a few m_P during slow roll inflation and with a potential typically of the form

$$V(\phi) = \Lambda^4 \left(\frac{\phi}{\mu}\right)^p . \tag{16}$$

These extremely simple potentials for inflation have been initially introduced in the context of so-called 'chaotic inflation'. Another typical potential in this category is the exponential potential,

$$V(\phi) = \Lambda^4 \exp\left(\phi/\mu\right), \tag{17}$$

which gives rise to power-law inflation, where the scale factor evolves like $a(t) \propto t^{1/\epsilon}$, with $\epsilon = \eta = (m_P/\mu)^2/2$.

This category of models has had a lot of success in the literature because of their computational simplicity. However, in general, they are not considered to be models which can be well motivated by particle physics. The reason is the following. The generic potential for a scalar field will contain an infinite number of terms,

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \frac{\lambda_3}{3}\phi^3 + \frac{\lambda_4}{4}\phi^4 + \sum_{d=5}^{\infty} m_P^{4-d}\phi^d, \tag{18}$$

where the non-renormalizable ($d > 4$) couplings λ_d are a priori of order 1. When the scalar field is of order of a few Planck masses, one has no control on the form of the potential, and all the non-normalizable terms must be taken into account in principle. In order to work with more specific forms for the potential, inflationary model builders tend to concentrate on models where the scalar field amplitude is small with respect to the Planck mass.

3.2. Spontaneous symmetry breaking models ($\eta < 0 < \epsilon$)

This type of models is characterized by $V''(\phi) < 0$, and a typical potential can be written in the form

$$V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right]. \tag{19}$$

This can be interpreted as the lowest-order term in a Taylor expansion around the origin. Historically, this potential shape appeared in the so-called 'new inflation' scenario, which followed the initial version of inflation, dubbed 'old inflation', based on a first-order transition which was shown to never end at least if one wants enough inflation to solve the usual horizon problem of standard cosmology.

A particular feature of these models is that tensor modes are much more suppressed with respect to scalar modes than in the large-field models.

3.3. Hybrid models ($0 < \epsilon < \eta$)

A new category of models has appeared more recently. In the hybrid scenario, two scalar fields are taken into account. One scalar field is responsible for inflation and evolves toward a minimum with nonzero vacuum energy, while the second field is responsible for the end of inflation. The 'decoupling' between the inflaton and the end of inflation allows a richer range of possibilities.

Hybrid inflation potentials, which frequently appear in supersymmetric models, are characterized by $V''(\phi) > 0$ and $0 < \epsilon < \eta$. As far as the spectrum of perturbations is considered, one needs only the shape of the effective potential of the inflaton field during inflation, which can be described in the form

$$V(\phi) = \Lambda^4 \left[1 + \left(\frac{\phi}{\mu}\right)^p\right]. \tag{20}$$

Once more, this potential can be seen as the lowest order in a Taylor expansion around the origin. The value ϕ_N of the scalar field as a function of the number of e-folds before the end of inflation is not determined by the above potential and, therefore, (ϕ_N/μ) can be considered as a freely adjustable parameter. A characteristic feature of hybrid models is that they can lead, in contrast to the two previous categories, to a blue spectrum ($n_s > 1$), although there are also models of hybrid inflation giving a red spectrum.

4. The importance of CMB polarization

Although this section is somehow related to the previous one, it may be useful to devote a special section to the subject of the CMB polarization, the measurement of which might open a qualitative new window on the early universe. Before discussing its connection with the primordial universe, in particular within the inflationary paradigm, let us first review the basic formalism used to describe the polarization [15].

The linear polarisation of the CMB can be described by a (two-index) tensor field on the two-sphere, in the same way as the CMB temperature is described by a scalar field on the two sphere. This tensor field can be decomposed onto a basis of tensor spherical harmonics, themselves separable into the electric-type tensor harmonics

$$Y^E_{(lm)ab} = N_l \left(Y_{(lm);ab} - \frac{1}{2}\gamma_{ab} Y_{(lm);c}^{\ \ ;c} \right), \tag{21}$$

and the magnetic-type tensor harmonics

$$Y^{B}_{(lm)ab} = \frac{N_l}{2}\left(Y_{(lm);ac}\varepsilon^{c}_{\ b} + Y_{(lm);bc}\varepsilon^{c}_{\ a}\right), \tag{22}$$

with the normalisation constant $N_l = \sqrt{2(l-2)!/(l+2)!}$, and where γ_{ab} is the metric on the two-sphere and ε_{ab} the associated antisymmetric tensor. In analogy with the multipole coefficients for the temperature defined in (1), which will be now denoted a^T_{lm}, one can define the electric multipole coefficients a^E_{lm} and the magnetic multipole coefficients a^B_{lm}. As a consequence, there will be in total four angular power spectra. In addition to the temperature power spectrum C^T_l defined in (2), one finds the electric and magnetic power spectra,

$$C^E_l = \langle |a^E_{lm}|^2\rangle, \quad C^B_l = \langle |a^B_{lm}|^2\rangle, \tag{23}$$

and the cross-correlation spectrum

$$C^C_l = \langle a^T_{lm} a^{E*}_{lm}\rangle. \tag{24}$$

The two other cross correlations one could envisage, E with B or T with B, automatically vanish for reasons of symmetry.

The importance of CMB polarization concerning the physics of the early universe relies on the fact that scalar fluctuations can produce only E-type polarization and no B-type polarization. A measurement of the B-polarization could therefore be interpreted as the detection of primordial gravitational waves. There are unfortunately a few problems in this attractive perspective. Obviously, the first difficulty is the smallness of the signal: the CMB is expected to be polarized only at the $5-10\%$ level on small angular scales, and even less on large angular scales. Detection of the CMB polarization is thus in itself a technological challenge. Another problem concerns the interpretation of a positive signal: not only primordial gravitational waves but also foreground emission, or any process with Faraday rotation, will generate B-type polarization. In principle, these contaminants could be substracted upon use of multi-frequency observations, but this makes the objective of actually measuring the amount of primordial waves still more remote.

5. More exotic possibilities

So far, we have presented the most consensual picture for the early universe with its relation to actual observations. In this sense, the slow-rolling single field inflationary scenario represents today the *minimal standard model of the early universe*. Within this perspective, the future path of research is well paved: with the constant refinement of CMB measurements expected for the forthcoming years, the constraints on the various parameters describing the slow-roll regime (and even beyond the slow-roll approximation) will be tighter and tighter, thus excluding more and more inflationary scenarios.

Of course, the early universe cosmologists have been trying constantly to explore alternative avenues. A long-standing opponent to the inflationary paradigm has been the formation of structure seeded by topological defects, which are predicted to be produced in Grand Unified Theories. The recent CMB data have shown that these models cannot explain, by themselves, the observed signal. Another interesting idea is the pre-big-bang scenario [16], trying to make the connection between string theory and the early universe. This scenario unfortunately suffers from two weaknesses: the transition between the pre-big-bang and the post-big-bang, where the physics is not

under control so far; and the fact that it does not seem to give a nearly scale-invariant scalar fluctuation spectrum.

In the exploration of more exotic scenarios, one must distinguish two types of approaches: one consists in considering more refined versions of the simplest model, thus adding more degrees of freedom than usually necessary to fit the data. This attitude is useful for two reasons: for exploring the robustness of the main model (i.e. this is in some sense a study of the degeneracy among early universe models in the neighbourhood of the reference model); for studying the possibilities of obtaining from the cosmological data some extra information which simply does not exist in the main model. Another, more radical approach, consists in trying to find completely new types of scenario, and see if they can reproduce the impressive successes of the inflationary scenario. We will now give one example for each of these two approaches.

5.1. Isocurvature perturbations

In the single field inflationary picture, the cosmological fluctuations produced during inflation are necessarily *adiabatic*, i.e. the relative composition in the various cosmological species is the same for the pertubations as for the background, because all species have the same origin: the unique scalar field. One must however keep in mind the possibility of *isocurvature* perturbations, defined in the case of two species X and Y by the non-vanishing quantity

$$S_{X,Y} = \frac{\delta n_X}{n_X} - \frac{\delta n_Y}{n_Y}, \tag{25}$$

where $n_{X,Y}$ are the number densities of the species, the perturbation in the total energy density being zero. Although pure isocurvature primordial spectra have been shown to be incompatible with cosmological data, a small fraction of isocurvature primordial perturbations is allowed although strongly constrained.

What has recently renewed the interest in isocurvature perturbations is the richer range of possibilities if these isocurvature perturbations are *correlated* with the usual adiabatic perturbations. This correlation has been first illustrated [17] in a very simple model of double inflation with two free massive scalar fields, with the Lagrangian

$$\mathcal{L} = -\partial_\mu \phi_h \partial^\mu \phi_1 - \frac{1}{2} m_1^2 \phi_1^2 - \partial_\mu \phi_l \partial^\mu \phi_2 - \frac{1}{2} m_2^2 \phi_2^2. \tag{26}$$

The two scalar fields being uncoupled, their quantum fluctuations $\delta\phi_1$ and $\delta\phi_2$ are statistically independent. However both fields will in general contribute to the primordial adiabatic and isocurvature perturbations,

$$\Phi = A_1 \delta\phi_1 + A_2 \delta\phi_2, \quad S = B_1 \delta\phi_1 + B_2 \delta\phi_2, \tag{27}$$

where the A's and B's are background dependent coefficients. It turns out that in some region of the parameter space, Φ and S will be correlated. Allowing for correlation between isocurvature and adiabatic perturbations gives more freedom to play with the predicted CMB spectrum and has been more systematically studied in several recent works [18].

5.2. Brane cosmology

The idea of extra-dimensions has recently gone through a renewal with the hypothesis, suggested by recent developments in string theory, that ordinary matter is confined to a three-dimensional subspace, or *brane*, embedded in a higher dimensional spacetime or

bulk. In cosmology, particular attention has been devoted to five-dimensional models where the worldsheet of our Universe-brane is a hypersurface.

A striking result, obtained when solving the five-dimensional Einstein's equations $G_{AB} \equiv R_{AB} - Rg_{AB}/2 = \kappa^2 T_{AB}$, is that the matter content of the brane enters *quadratically* [19] in the Friedmann equations instead of linearly as in standard cosmology. In the case of an empty bulk, one would thus find a cosmological evolution incompatible with our understanding of nucleosynthesis. A way out has been found by applying the Randall-Sundrum idea [20] to cosmology [21, 22], i.e. considering an Anti-de Sitter bulk spacetime (with a negative cosmological constant Λ) and a tension in the brane. The (assumed) cancellation of Λ with the square of the brane tension σ leads to the new Friedmann equations [23, 22]

$$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{\rho^2}{2\sigma}\right), \tag{28}$$

where H is the Hubble parameter in the brane, ρ the cosmological energy density in the brane. And Newton's constant is related to the brane tension by $8\pi G = \kappa^4\sigma/6$. This equation gives the usual evolution in the low energy regime $\rho \ll \sigma$ and quadratic corrections in the high energy regime $\rho > \sigma$.

The next step is obviously the influence of extra-dimensions on the *cosmological perturbations* and their evolution, and thus try to make the link with a specific signature for the CMB predictions. Several pioneering works have developed formalisms to handle the cosmological perturbations for a brane-universe in a five-dimensional spacetime (see [24] and references therein). In the case of slow-roll inflation generated by a scalar field confined to the brane [25], one can compute explicitly the cosmological fluctuations generated during a quasi- de Sitter phase, both for the scalar spectrum [25] and the tensor spectrum [26]. However, for the subsequent radiation and matter dominated eras, the evolution of perturbations is much more complicated, except in the case of super-Hubble perturbations [27], because a quantitative analysis and therefore the determination of the CMB anisotropy spectrum depends on the specific distribution of gravitational waves in the bulk.

6. Conclusions

It is always extremely difficult to make predictions about the future of a scientific domain and I will not take this risk. The amount of information we will learn from the future CMB observations will depend on how close or how far they turn out to be with respect to the *canonical* version of the early universe model, that of an inflationary phase generated by a single field in slow-roll motion. To be too close or too far are probably the cases where the gain of information will be minimal for theorists. Obviously, being very close would be a tremendous success for the currently prefered model but would not bring any surprise. However, being too far might not provide so much information as well, unless it corresponds to the predictions of a model already considered. We would have to revise some of our ideas but it is usually easy for theorists to cook up a model, or even several, which will fit the data *a posteriori*. The most stimulating situation for cosmology, where the number of observations concerning the early universe is extremely limited, would occur if the new data roughly confirm the overall picture but add details that reveal some deviations from the canonical model. This is the best situation to learn something because this is the case where

one is most likely to interpret correctly the unexpected features of the data. Let us hope therefore than the next observations will put us in such a position.

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